T94/150
hep-ph/9501239

EFFECTIVE FIELD THEORIES FOR HADRONS AND NUCLEI*

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December 1994

ABSTRACT

Hadron structure and nuclear structure are discussed from the common ground of effective chiral Lagrangians modeling QCD at low energy. The topics treated are the chiral bag model in large N_c QCD, its connection to heavy-baryon chiral perturbation theory (HBχPT), the role of nonabelian Berry gauge connections for baryon excitations and the application of HBχPT to the thermal $n+p \to d+\gamma$ process and to the axial-charge transitions in heavy nuclei.

* Invited talk given at *Nishinomiya Yukawa-Memorial Symposium in Theoretical Physics*, October 27-28, 1994, Nishinomiya, Japan.

1 Introduction

The quarks that enter into nuclei and hence figure in nuclear physics are the u(p), d(own) and possibly s(trange) quarks. These are called "chiral quarks" since they are very light at the scale of strong interactions. Both the u and d quarks are less than 10 MeV, much less than the relevant scale which I will identify with the vector meson (say, ρ) mass ~ 1 GeV. The s quark is in the range of 130 to 180 MeV, so it is not quite light. In some sense, it may be classified as "heavy" as in the Skyrme model for hyperons but the success with current algebras involving kaons also indicates that it can be considered as chiral as the u and d are. In this talk, I will consider the s quark as both heavy – when I describe baryon excitations and light – when I describe kaon condensation.

If the quark masses are zero, the QCD Lagrangian has chiral symmetry $SU(n_f)\times SU(n_f)$ where n_f is the number of massless flavors. However we know that this symmetry in the world we are living in, namely at low temperature (T) and low density (ρ), is *spontaneously* broken to $SU(n_f)_V$ giving rise to Goldstone bosons denoted π, the pions for $n_f = 2$, the pions and kaons for $n_f = 3$. In nature, the quark masses are not strictly zero, so the chiral symmetry is explicitly broken by the masses, and the bosons are pseudo-Godstones with small mass. Again in the u and d sector, the pion is very light ~ 140 MeV but in the strange sector, the kaon is not so light, ~ 500 MeV. Nonetheless we will pretend that we have good chiral symmetry and rectify our mistakes by adding symmetry breaking terms treated in a suitable way.

The theme of this talk is then that most of what happens in nuclei are strongly controlled by this symmetry pattern. Indeed, it was argued many years ago[1] that chiral symmetry should play a crucial role in many nuclear processes, much more than confinement and asymptotic freedom – the other basic ingredients of QCD – would. More recently, it has become clear that much of what we can understand of the fundamental nucleon structure also follows from chiral symmetry and its breaking. This was also anticipated sometime ago[2, 3].

In this talk, I would like to tell you more recent and quite exciting new development in this line of work which suggests that the old idea, quite vague at the beginning, is becoming a viable model of QCD in many-body nuclear systems.

2 Nucleon Structure: The Chiral Bag in QCD

The chiral bag was formulated originally in a somewhat as hoc way based solely on chiral symmetry but there is a striking indication[4, 5] that it follows from a more general argument based on large N_c QCD where N_c is the number of colors. Let me discuss this as a model for nucleon structure.

When chiral symmetry is implemented to the bag model of the hadrons[3, 6], it was found necessary to introduce pion fields outside of the bag of radius R in which quarks are "confined." This is because otherwise the axial current cannot be conserved. Furthermore, it was discovered[7] that to be consistent with non-perturbative structure, the pion field takes the form of the skyrmion configuration with a fractional baryon charge residing in the pion cloud. This implied that the quarks are not strictly confined in the sense of the MIT bag but various charges leak out. It became clear that the bag radius is not a physical variable. That physics should not depend upon the size of the bag has been formulated as a "Cheshire Cat Principle" (CCP). In fact the CCP may be stated as a gauge principle[8] with the bag taken as a gauge fixing. What this meant was that the skyrmion is just a chiral bag whose radius is "gauge-chosen" to be shrunk to a point.

The recent development[4, 5] is closer to the core of QCD. In large N_c QCD, meson-meson interactions become weak but meson-baryon Yukawa interactions become strong going like $N_c^{1/2}$. In this limit, the baryon is heavy and hence can be treated as a static source localized at the origin. Other interactions, such as mass splittings etc. are down by a factor of N_c. Thus we have to add to the usual current algebra Lagrangian of $O(N_c)$, L_{ca}, a term of the form[5]

$$\delta L = 3g_A \delta(\vec{x}) X^{ia} A^{ia}(x) \tag{1}$$

where X^{ia} is the baryon axial current in the large N_c limit and A^{ia} is the pion axial current. It is found that summing an infinite class of Feynman diagrams in the leading N_c order corresponds to solving coupled classical field equations given by the leading order Lagrangian $L_{ca} + \delta L$. This produces a baryon source coupled with a classical meson cloud, with quantum corrections obtained by performing semiclassical expansion around the classical meson background. This is precisely the picture described by the chiral bag[7].

Two aspects of this result are important:

- It is conjectured – and seems highly plausible – that there is a line of UV fixed points in the large N_c renormalization group flow of the parameters of the Lagrangian[4]. The bag radius can be one of those parameters. If correct, one may formulate CCP in terms of the "fixed line."

- The m_π^3 (or $m_q^{3/2}$) (where m_π is the pion mass and m_q the quark mass) non-analytic correction to the baryon mass that is found in the classical solution is identical to a loop correction in chiral perturbation theory[5]. This makes a direct link between the chiral bag and chiral perturbation theory (χPT) to a higher chiral order. We shall exploit this fact later for nuclear processes and also for Goldstone boson condensations in dense hadronic matter.

3 Baryon Excitations and Berry Potentials

The chiral bag in large N_c limit can be identified with the skyrmion model, and it describes the baryon "ground state" with the semiclassical configuration in the hedgehog form in flavor $SU(2)$. Collective rotation built on the hedgehog gives the spectrum with excitations described by $J = I$ up to $J_{max} = N_c/2$. I would like to discuss how one can describe other baryonic excitations with $J \neq I$ as well as excitations involving change of flavor away from the light-quark (u and d) space. It will be seen that such baryon excitations can also be described based on generic symmetry considerations, this time in terms of generalized Berry potentials. This line of reasoning was developed in [9, 10] where all other references can be found.

Consider the chiral bag picture. The excitations we wish to consider can be classified chiefly as follows. One class of excitation involves quark excitations inside the chiral bag from the lowest hedgehog state with the grand spin $K = 0^+$ where $\vec{K} = \vec{J} + \vec{I}$ to $K \neq 0$ excited quark orbitals. This is an excitation within the $SU(2)$ flavor space. A different class of excitation involves changes of flavor, corresponding to an excitation from the $K = 0^+$ orbital to a strange (s), charm (c) or bottom (b) orbital. One quark excitation of either class corresponds to making a "particle-hole" (p-h) state in the many-body theory language and such excitations will be coupled on the bag surface with the corresponding mesons living outside of the bag, *e.g.*, the K-meson, D-meson, B-meson respectively. For simplicity we will assume that the frequency ω_{vib} associated with the p-h excitation (or vibration for short) is much greater the rotational frequency ω_{rot} for the collective rotation of the bag. We shall identify the vibration as the "fast" degree of freedom and the rotation as the "slow" degree of freedom. The situation then presents a case susceptible to the Born-Oppenheimer approximation and the idea is to "integrate out" the fast degree of freedom in favor of the slow degree of freedom, leaving the imprint of the fast degree of freedom in the space of the slow variable. This is a very generic situation and can be studied with a simple quantum mechanical system.

3.1 Analogy with diatomic molecules

Following discussion in Ref.[11], we consider a generic Hamiltonian describing a system consisting of the slow ("nuclear") variables $\vec{R}(t)$ representing the dumb-bell diatom (with $\vec{P}$ as conjugate momenta) and the fast ("electronic") variables $\vec{r}$ (with $\vec{p}$ as conjugate momenta) coupled through a potential $V(\vec{R}, \vec{r})$

$$H = \frac{\vec{P}^2}{2M} + \frac{\vec{p}^2}{2m} + V(\vec{R}, \vec{r}) \tag{2}$$

where we use the capitals for the slow variables and lower-case letters for the fast variables. To describe the symmetry of the system, let $\vec{N}$ be the unit vector along the internuclear

axis and define the quantum numbers

$$\begin{aligned}\Lambda &= \text{eigenvalue of } \vec{N}\cdot\vec{L}\\ \Sigma &= \text{eigenvalue of } \vec{N}\cdot\vec{S}\\ \Omega &= \text{eigenvalue of } \vec{N}\cdot\vec{J} = |\Lambda+\Sigma|,\end{aligned} \tag{3}$$

so Λ, Σ, Ω are the projections of the orbital momentum, spin and total angular momentum of the electron on the molecular axis, respectively. For simplicity we focus on the simple case of $\Sigma = 0$, $\Lambda = 0, \pm 1$. The $\Lambda = 0$ state is referred to as Σ state and the $\Lambda = \pm 1$ states are called π, a degenerate doublet. We are interested in the property of these triplet states, in particular in the symmetry associated with their energy splittings.

Upon integrating out the fast electronic degrees of freedom using the usual adiabatic approximation, we can write the resulting Lagrangian in the form

$$L^{eff}_{nm} = \frac{1}{2}M\dot{\vec{R}}(t)^2\delta_{mn} + \vec{A}_{mn}[\vec{R}(t)]\cdot\dot{\vec{R}}(t) - \epsilon_m\delta_{mn} \tag{4}$$

where

$$A_{m,n} = i < m|\nabla|n > \tag{5}$$

is a nonabelian Berry potential with the indices m, n labeling the triplet states. This can be rewritten in non-matrix form (dropping the trivial electronic energy ϵ)

$$\mathcal{L} = \frac{1}{2}M\dot{\vec{R}}^2 + i\theta_a^\dagger(\frac{\partial}{\partial t} - i\vec{A}^\alpha T^\alpha_{ab}\cdot\dot{\vec{R}})\theta_b \tag{6}$$

where we have introduced a Grassmannian variable θ_a as a trick to avoid using the matrix form of (4) and $\mathbf{T}^\alpha$ is a matrix representation in the vector space in which the Berry potential lives satisfying the commutation rule

$$[\mathbf{T}^\alpha, \mathbf{T}^\beta] = if^{\alpha\beta\gamma}\mathbf{T}^\gamma. \tag{7}$$

By a suitable gauge transformation, the Berry potential can be put in the form that shows its structure as a 't Hooft-Polyakov monopole

$$\vec{\mathbf{A}} = (1-\kappa)\frac{\hat{R}\times\vec{I}}{R} \tag{8}$$

with the field strength tensor

$$\vec{\mathbf{B}} = -(1-\kappa^2)\frac{\hat{R}(\hat{R}\cdot\mathbf{I})}{R^2} \tag{9}$$

where

$$\kappa(R) = \frac{1}{\sqrt{2}}|\langle\Sigma|L_x - iL_y|\pi\rangle|. \tag{10}$$

Upon quantizing the theory, one gets the "hyperfine" spectrum given by the Hamiltonian

$$\Delta H = \frac{1}{2MR^2}(\vec{L}_o + (1-\kappa)\vec{I})^2 - \frac{1}{2MR^2}(1-\kappa)^2(\vec{I}\cdot\hat{R})^2 \tag{11}$$

where L_0 is the angular momentum of the dumb-bell and I the angular momentum lodged in the gauge field (inherited from the electronic sector). The conserved angular momentum is

$$\vec{L} = \vec{L}_o + \vec{I}, \tag{12}$$

so

$$\Delta H = \frac{1}{2MR^2}(\vec{L} - \kappa\vec{I})^2 - \frac{1}{2MR^2}(1-\kappa)^2 \tag{13}$$

where $(\vec{I}\cdot\hat{R})^2 = 1$ has been used. Here $(1-\kappa)$ is a "monopole charge" and is clearly not quantized since the value of κ is not an integer. Indeed physics lies in the value of κ as defined in (10).

Let us consider the two extreme cases. The π doublet are degenerate but the Σ state is split from the π according to the separation of the dumb-bell R. As $R \to 0$, the energy of the Σ state becomes much higher than the doublet, as a consequence of which $\kappa \to 0$ and the spectrum tends to that of the Dirac monopole with the monopole charge $g = 1 - \kappa = 1$. If on the other hand, $R \to \infty$, then the energy of the Σ state becomes degenerate with the doublet and hence $\kappa \to 1$. In this case, the angular momentum stored in the gauge field gets decoupled from the system, that is, the spectrum becomes independent of the angular momentum stored in the Berry potential. The consequence is that the electronic rotational symmetry is restored in that limit. This restoration of the rotational symmetry will have an analog in the heavy-quark symmetry discussed below.

3.2 Heavy-quark baryons

The above reasoning can be applied almost immediately to the excitation spectrum of heavy-quark baryons. (One can apply it also to the excitations of light-quark baryons but the adiabatic approximation is not very good, and nonadiabatic corrections cannot be ignored. See [9] on this matter.) Let Φ be the "heavy" mesons K, D, B which are the analog to the electronic excitation in the diatomic case. We then identify the moment of inertia of the rotating soliton $\mathcal{I}$ with MR^2, the hyperfine coefficient c with the "monopole charge" $(1-\kappa)$. Let the angular momentum stored in the soliton be denoted by J_{sol} and the angular momentum stored in the gauge field inherited from the heavy mesons be J_Φ. Then the hyperfine spectrum takes the form

$$\Delta H = \omega_\Phi + \frac{1}{2\mathcal{I}}\left(\vec{J}_R + c_\Phi \vec{J}_\Phi\right)^2 + \cdots \tag{14}$$

Figure 1: Spectra for strange and charmed hyperons predicted in the model Eq.(14) compared with the quark model. The fit parameters are $\omega_K = 223$ MeV, $c_K = 0.62$, $\omega_D = 1418$ MeV, $c_D = 0.14$.

where I have put the "vibrational" frequency ω for the heavy meson Φ bound in the soliton as described above. This is a result that follows from the general consideration of the Berry structure. It was obtained by Callan and Klebanov[12] for strange hyperons using a different method.

Given the formula with the moment of inertia $\mathcal{I}$ determined in the $SU(2)$ soliton sector, we may now take ω_Φ and c_Φ as parameters for each Φ and fit the spectra. QCD would eventually predict those quantities. But for our purpose it is not essential how one gets them.

The resulting fit is given in Fig.1 for strange and charm hyperons. One can obtain a similar fit for the bottom baryons.

From general consideration[13] of large N_c behavior and heavy-quark symmetry of QCD, one expects that the constant c should behave

$$c_\Phi \sim c/m_\Phi \tag{15}$$

where c is an $O(N_c^0 m_\Phi^0)$ constant. The fit indeed shows this with $c \sim 262$ MeV. In the limit $m_\Phi = 0$, the Berry potential argument shows[9] that $c_\Phi = 0$. This is the heavy-quark limit resembling closely the $R = \infty$ limit of the diatomic molecule.

4 Chiral Perturbation Theory for Nuclei and Nuclear Matter

We have seen that in describing the structure of an *elementary* baryon, the large N_c chiral bag model can be mapped to chiral perturbation theory in terms of baryons and

mesons given as local fields[5]. We will now take up the same effective chiral Lagrangian and apply it to many-body systems, *i.e.*, nuclei and nuclear matter.

4.1 Nuclear matter as a Fermi-liquid fixed point

There are two classes of physical processes we are interested in. One is the ground-state property of nuclei and nuclear matter from the point of view of effective chiral Lagrangians. Given an understanding of this, we would like to be able to describe the state of matter as we change the density and /or temperature.

The other is to understand nuclear force and nuclear response functions in terms of chiral Lagrangians.

The description of these two classes of process would require a field theory that can describe simultaneously normal nuclear matter and phase transitions therefrom. The most relevant ingredient of QCD that is needed here is spontaneously broken chiral symmetry. For the first, we will be specifically interested in chiral $SU(3) \times SU(3)$ symmetry since as we shall see, strangeness is involved. In order to address this problem, we need to start from a realistic effective chiral Lagrangian, obtain a nuclear matter of the right properties from it and then determine whether a phase change occurs. For the second, the situation is a bit different and this "self-consistency" problem can be circumvented in a sense explained below.

At present, we do not have a good description of nuclear matter (and nuclei) starting from a chiral Lagrangian. There are various suggestions and one promising one is that nuclear matter arises as a solitonic matter from a chiral effective action, a sort of chiral liquid[14] resembling Landau Fermi liquid. The hope is that the resulting effective action would look like Walecka's mean-field model. There is as yet no convincing derivation along this line. In the work reported here, we will have to assume that we have a nuclear matter that comes out of an effective chiral action. Given such a ground state, we would like to study fluctuations along various flavor channels and study both nuclear response functions to slowly varying external fields and possible instabilities under extreme conditions leading to possible phase transitions. We are therefore assuming that we can get the properties of normal nuclear matter (and nuclei) from phenomenology, that is, that nuclear matter is a Fermi-liquid fixed point[15, 16].

In principle, a precise knowledge of this ground state from a chiral effective Lagrangian at a nonperturbative QCD level would allow us to determine the coefficients that appear in the effective Lagrangian with which to describe fluctuations around the soliton background – i.e., the Fermi liquid –and with which we could then compute all nuclear response functions. At present such a derivation does not exist. In a recent paper by Brown and the author (BR91)[29], it is assumed that in medium at a matter density $\rho \sim \rho_0$, the *nuclear* effective

field theory can be written in terms of the medium-dependent coupling constants $g^\star$ and masses of hadrons $m^\star$ while preserving the free-space structure of a sigma model. This leads to the so-called Brown-Rho scaling. In [29], the nonlinear sigma model implemented with trace anomaly of QCD is used to arrive at the scaling law. The precise way that this scaling makes sense is elaborated by Adami and Brown[32] and in the review (BR94)[33]. I return to this matter below.

4.2 Chiral counting

To describe nuclei and nuclear matter, we need an effective chiral Lagrangian involving baryons as well as Goldstone bosons. When baryons are present, χPT is not as firmly formulated as when they are absent[17]. The reason is that the baryon mass m_B is $\sim \Lambda_\chi \sim 1$ GeV, the chiral symmetry breaking scale. It is more expedient, therefore, to redefine the baryon field so as to remove the mass from the baryon propagator

$$B_v = e^{im_B\gamma\cdot v\; v\cdot x} P_+ B \tag{16}$$

where $P_+ = (1+\gamma\cdot v)/2$ and write the baryon four-momentum

$$p_\mu = m_B v_\mu + k_\mu \tag{17}$$

where k_μ is the small residual momentum indicating the baryon being slightly off-shell. When acted on by a derivative, the baryon field B_v yields a term of $O(k)$. Chiral perturbation theory in terms of B_v and Goldstone bosons $(\pi\cdot\lambda/2)$ is known as "heavy-baryon (HB) χPT"[18]. The HBχPT consists of making chiral expansion in derivatives on Goldstone boson fields, ∂_M/Λ_χ, and on baryon fields, ∂_B/m_B, and in the quark mass matrix, $\kappa\mathcal{M}/\Lambda_\chi^2$. In the meson sector, this is just what Gasser and Leutwyler did for $\pi\pi$ scattering. In the baryon sector, consistency with this expansion requires that the chiral counting be made with $B^\dagger(\cdots)B$, not with $\bar{B}(\cdots)B$. This means that in medium, it is always the baryon density $\rho(r)$ that comes in and *not* the scalar density $\rho_s(r)$.

Following Weinberg[19], we organize the chiral expansion in power Q^ν where Q is the characteristic energy/momentum scale we are looking at ($Q << \Lambda_\chi$) and

$$\nu = 4 - N_n - 2C + 2L + \sum_i \Delta_i \tag{18}$$

with the sum over i running over the vertices that appear in the graph and

$$\Delta_i = d_i + \frac{1}{2}n_i - 2. \tag{19}$$

Here ν gives the power of small momentum (or energy) for a process involving N_n nucleon lines, L number of loops, d_i number of derivatives (or powers of meson mass) in the ith

vertex, n_i number of nucleon lines entering into ith vertex and C is the number of separate connected pieces of the Feynman graph. In the absence of external gauge fields, chiral invariance requires that $\Delta_i \geq 0$, so that the leading power is given by $L = 0$, $\nu = 4 - N_N - 2C$. If we are interested in nuclear responses to external electroweak fields, then $\Delta_i \geq -1$.

4.3 Nuclear forces from chiral Lagrangians

The question as to how much of nuclear forces can be understood starting from chiral Lagrangians was recently addressed by Weinberg[19] and by Ordóñez, Ray and van Kolck[20]. The authors of [20] studied a chiral Lagrangian consisting of nucleons, Δ's and pions applying it to the nucleon-nucleon potential to the chiral order $\nu = 3$ corresponding to $N_n = 2$, $C = 1$, $L = 1$ and $\Delta_i = 1$ in (18). Using a cut-off regularization with a cut-off of order $\Lambda \sim 3.9\text{fm}^{-1}$ and fitting the resulting 26 parameters including counter terms to $I = 0$ np and $I = 1$ pp phase shifts to ~ 100 MeV and to deuteron properties, they were able to reproduce the global experimental data. The import of this work is not that it can provide a better potential than what is currently available in the phenomenological approach but that nuclear potential can be understood at least at low energy $E \lesssim 100$ MeV from the chiral symmetry point of view.

Effective chiral Lagrangians can also make interesting statements about nuclear forces in many-body systems, in particular about many-body forces[19] and exchange currents[21].

If energy or momentum scale probed Q is much less than the typical chiral scale $\Lambda_\chi \sim 1$ GeV, then in many-body systems, one can use static approximation for the pion exchange. In this case, to the order of chiral expansion that we can actually use at the moment – which corresponds to next-to-next-to-leading (N^2L) order, three-body forces and currents are exactly canceled with higher-body forces and currents further suppressed. This justifies the conventional practice in nuclear physics of ignoring many-body forces and currents. Of course in higher energy scale at which higher chiral orders are required, many-body forces and currents will have to be included. This aspect will be clearly relevant in the future experiments at CEBAF where multi-GeV energy and momentum transfers will be involved.

4.4 Exchange currents

Chiral Lagrangians have recently scored an impressive success in describing exchange vector and axial vector currents at low momentum transfer. In applying chiral Lagrangians to nuclear response functions, one has to recognize that while nuclear interactions sample whole range of distances entering into strong interactions, χPT is applicable only at sufficiently large distance scales. Thus the most profitable way of exploiting χPT is to calculate the appropriate amplitude embedded in the graph sandwiched between initial and final

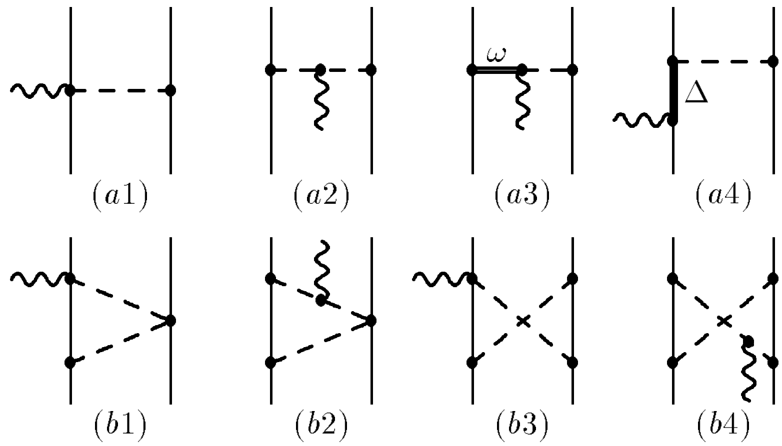

Figure 2: The Feynman graphs which contribute to the two-body vector current. The wiggly line is the vector current, the broken line the pion and the solid line the nucleon. The "generalized tree graphs" with the vertices renormalized by loops are drawn in (a) and the two-pion-exchange graphs in (b). In Figure (a), the various one loop graphs appearing in $\pi\mathcal{V}NN$ vertex are entirely saturated by the resonance-exchange tree graphs ($(a3)$ and $(a4)$). Equivalent but topologically different graphs are not drawn. Graphs that do not contribute in HFF as well as those that have zero measure in nuclei are also not shown.

nuclear interactions that are described by realistic nuclear potentials. This is the strategy that has been used since a long time[22].

Since for slowly varying external field, three-body, four-body ... currents can be ignored to the chiral order we will consider, we can focus on two-nucleon processes. Thus in the counting rule (18), we will have $N_n = 2$, $C = 1, 2$ and $\Delta_i \geq -1$. Thus a single-particle operator will have $\nu = -3$ at the leading order (with $C = 2$), the leading (tree) two-body operator $\nu = -1$ with one-loop corrections coming in at $\nu = +1$. Here we will calculate to one-loop order, hence our χPT corresponds to N^2L order.

The most interesting cases to consider are the axial charge operator A_0^i and the isovector magnetic moment operator μ coming from the vector current $\vec{V}^3$. In both cases, the single-particle operator has an additional $1/m_N$ suppression factor, so its chiral order is $\nu = -2$ instead of -3. Now to N^2L order in HBχPT, the multitude of graphs reduce to a handful of them. For instance, for the magnetic moment operator, there are eight non-vanishing two-body graphs as given in Fig.2. For the axial-charge operator, there is further reduction as I will describe below.

4.4.1 *Thermal np capture*

We first consider the most classic nuclear process[23]

$$n + p \to d + \gamma \tag{20}$$

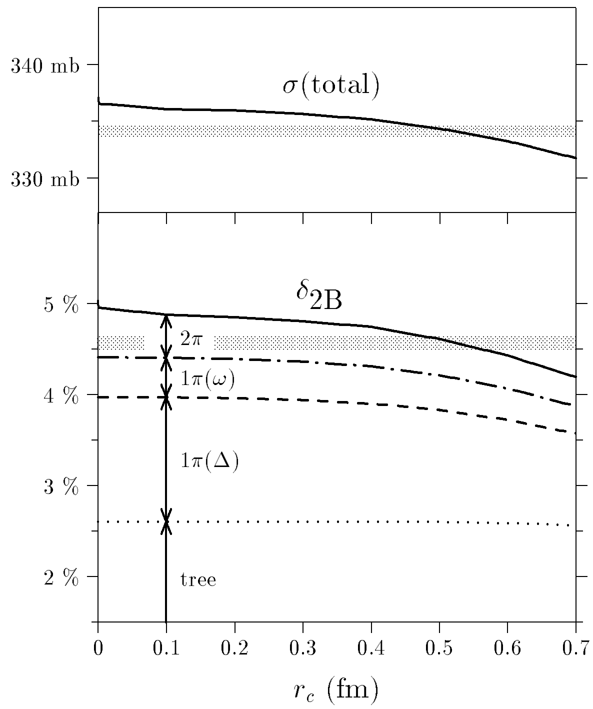

Figure 3: Total capture cross section $\sigma_{\rm cap}$ (top) and δ's (bottom) vs. the cut-off r_c. The solid line represents the total contributions and the experimental values are given by the shaded band indicating the error bar. The dotted line gives $\delta_{\rm tree}$, the dashed line $\delta_{\rm tree} + \delta_{1\pi}^{\Delta}$, the dot-dashed line $\delta_{\rm tree} + \delta_{1\pi} = \delta_{\rm tree} + \delta_{1\pi}^{\Delta} + \delta_{1\pi}^{\omega}$ and the solid line the total ratio, $\delta_{\rm 2B}$.

which was first explained by Riska and Brown[24]. The result of Riska and Brown has recently been reproduced in χPT to N^2L order. The power of χPT is that to N^2L order, the eight graphs of Fig.2 are all there is to calculate. This clearly goes a considerable distance toward QCD in comparison to what was achieved in [22]. To this order, the dominant terms are the "generalized tree" graphs Fig.2(a1)-(a4) with renormalized coupling constants. The graphs (a1) and (a2) are the leading tree contribution (called "tree") and the graphs (a3) and (a4) are $O(Q)$ (or $O(Q^3)$ relative to the leading term) counter-term contributions that are saturated by the resonances (called "1π" in Fig.3). There are no other operators coming from the one-loop radiative correction to the vector-N-N-π vertex. The remaining two-pion one-loop graphs (b1)-(b4) (called "2π") makes a small contribution, less than 0.6% of the single-particle term.

In Fig.3 is given the result obtained with the most recent Argonne v_{18} potential of Wiringa, Stoks and Schiavilla[25] (which is fit 1787 pp and 2514 np scattering data in the range 0-350 MeV with a χ^2 of 1.09 and describes the deuteron properties accurately) compared with the experimental value[26]. The impulse operator predicts $\sigma_{imp} = 305.6$ mb, about 9.6% less than the experimental value $\sigma_{exp} = 334.2 \pm 0.5$ mb. Using a short-range

correlation cutoff $0 < r_c \lesssim 0.7$ fm to screen short-range interactions (which χPT cannot handle), the theoretical prediction comes out to be $\sigma_{th} = 334 \pm 3$ mb in a beautiful agreement with the experiment. Figure 3(b) shows how each term contributes to the amplitude relative to the impulse.

4.4.2 *Axial-charge transitions*

The axial-charge transition

$$A(J^+) \leftrightarrow B(J^-), \quad \Delta T = 1 \tag{21}$$

in nuclei is known to be enhanced compared with the impulse approximation prediction. The enhancement can be as much as 100% in heavy nuclei as Warburton has shown[27]. This can be understood very simply in terms of chiral Lagrangians[21, 28].

In the case of the axial charge operator, there is further reduction in the number of diagrams from the vector-current case: Figs.2(a2) and (b2) are absent by G-parity, (a4) is suppressed, (a3) has a ρ-meson replacing the ω-meson. There is however one term which is absent in the case of the magnetic moment operator. This has to do with the fact that in the graph (a3) it is the ρ meson that contributes. The ρ is coupled to two pions and it can give a one-loop vertex correction to the $A_0^i \pi NN$ vertex in contrast to the vertex $\vec{V}\pi NN$ which receives loop corrections only at two-loop order.

To state the result of the calculation[21, 28], we write the nuclear matrix element of the axial-charge operator as

$$M^{th} = M_1 + M_2 \tag{22}$$

with

$$M_2 = M_2^{tree}(1+\delta) \tag{23}$$

where the subscript represents the $n = 1, 2$-body operator and the "tree" corresponds to Fig.2(a1) (with renormalized coupling constants). It is found that to a good accuracy and almost independently of mass number[21, 28],

$$\delta \lesssim 0.1. \tag{24}$$

Again as in the electromagnetic case, the tree contribution dominates. This dominance of the soft-pion process in the cases considered was called "chiral filter phenomenon." Calculation of the soft-pion term with realistic wave functions[28] gives a large ratio

$$M_2^{tree}/M_1 \sim 0.6 \; - \; 0.8 \tag{25}$$

enough to explain the experimental value[27]

$$\frac{M^{exp}}{M_1} \sim 1.6 \; - \; 2. \tag{26}$$

This offers another indication that the pion cloud plays a crucial role in nuclear processes.

5 "Swelled" Hadrons in Medium

The effective chiral Lagrangian I used so far is a Lagrangian that results when the degrees of freedom lying above the chiral scale Λ_χ are eliminated by "mode integration." As one increases the matter density or heats the matter, the scale changes, so we can ask the following question: If a particle moves in a background with a matter density ρ and/or temperature T, what is the effective Lagrangian applicable in this background? One possible approach is to take a theory defined at zero T and zero ρ and compute what happens as T or ρ increases. This is the approach nuclear physicists have been using all along. However now we know that the major problem with QCD is that the vacuum is very complicated and we are not sure that by doing the standard approach we are actually describing the vacuum correctly as T or ρ goes up. Since the quark condensate $\langle\bar{q}q\rangle$ is a vacuum property and it changes as one changes T or ρ, it may be more profitable to change the vacuum appropriate to the given T or ρ and build an effective theory built on the changed vacuum. This is the idea of Brown and Rho[29] in introducing scaled parameters in the effective Lagrangian.

If the quarks are massless, then the QCD Lagrangian is scale-invariant but quantum mechanically a scale is generated giving rise to the trace anomaly. In the vacuum, we have in addition to the quark condensate $\langle\bar{q}q\rangle$ the gluon condensate $\langle G_{\mu\nu}G^{\mu\nu}\rangle$. We can associate a scalar field χ to the G^2 field as $G^2 \sim \chi^4$ and introduce the χ field into the effective Lagrangian to account for the conformal anomaly of QCD. The χ field can be decomposed roughly into two components, one "smooth" low frequency component and the other "non-smooth" high-frequency component. The former can be associated with 2-π, 4-π etc. fluctuations and the latter with a scalar glueball. For low-energy processes we are interested in, we can integrate out the high-energy component and work with the low-energy one. In dense matter, the low-energy component can be identified with a dialton as suggested by Beane and van Kolck[30]. Given this identification, one can show that the BR scaling follows from generic chiral Lagrangians as shown by Kusaka and Weise[31]. How this can be done in an unambiguous way is explained in [32, 33]. The outcome of this operation is that one can write the same form of the effective Lagrangian as in free space with the parameters of the theory scaled as

$$\frac{f_\pi^*}{f_\pi} \approx \frac{m_V^*}{m_V} \approx \frac{m_\sigma^*}{m_\sigma} \approx \cdots \equiv \Phi(\rho) \tag{27}$$

The nucleon effective mass scaled somewhat differently

$$\frac{m_N^*}{m_N} \approx \sqrt{\frac{g_A^*}{g_A}}\frac{f_\pi^*}{f_\pi}. \tag{28}$$

In these equations the asterisk stands for in-medium quantity. Now in the skyrmion model, at the mean-field level, $g_A^\star$ does not scale, so the nucleon will also scale as (27). It turns out that the pion properties do not scale; the pion mass remains unchanged in medium at low T and ρ. Thus if one of the ratios in (27) is determined either by theory or by experiment, then the scaling is completely defined. At densities up to nuclear matter density, the scaling is roughly

$$\begin{aligned} \Phi(\rho) &\approx 1 - a(\rho/\rho_0), \\ a &\approx 0.15 - 0.2 \end{aligned} \tag{29}$$

where ρ_0 is the normal nuclear matter density.

Now given the Lagrangian with the scaled parameters, we can go on and do loop corrections. One of the first things that one finds is that the $g_A^\star$ gets reduced to ~ 1 in nuclear matter from 1.26 in free space. So one would have to do the whole thing in a consistent way. However the point is that most of the processes in nuclear physics are dominated by tree-order diagrams and this means that the effective Lagrangian with the scaled parameters should be predictive without further corrections. Indeed this has been what has been found. In a recent paper, Brown, Buballa, Li and Wambach[34] use this "BR scaling" to explain simultaneously the new deep inelastic muon scattering experiment and Drell-Yan experiments.

A set of rather clear predictions has been made in this theory[32, 10].

6 Kaon Condensation

Since I am going to discuss this matter in detail in the Kyoto Workshop, I shall be rather brief on this matter.

In a way analogous to describing the baryon mass formula in terms of the large N_c chiral bag (or skyrmions) and χPT in heavy-baryon formalism, one can treat kaon condensation in two ways: One in the skyrmion model and the other in HBχPT. The two approaches give about the same answer.

6.1 The Callan-Klebanov Skyrmion on a Hypersphere

Callan and Klebanov[12] suggested in a beautiful paper in 1985 that in dense medium, the "effective mass" of the $\overline{K}$ meson bound in an $SU(2)$ skyrmion could decrease and when

it reaches zero, kaons would condense. This suggestion was examined by Forkel *et al.*[35] by putting the Callan-Klebanov skyrmion on a hypersphere following the idea of Manton[36] that the chiral and deconfinement phase transition(s) could be simulated by putting a single skyrmion on a hypersphere and by shrinking its radius. Forkel *et al.* were interested in the situation where the kaon mass vanished as would be relevant in heavy-ion collisions. This was however found to be impossible except at infinite density or unless the kaon mass "ran" down as a function of density. But as shown above, while the condition that the kaon mass go to zero may be required for condensation in heavy-ion physics, this is not what is needed in compact-star matter: It is enough that the mass decrease to the electron chemical potential μ_e.

A more realistic calculation taking this chemical potential into account was made recently by Westerberg[37]. For the parameters of the Skyrme Lagrangian fit to hyperon spectra, the critical density comes out to be

$$\rho_c = 0.595\,\mathrm{fm}^{-3} \simeq 3.5\,\rho_0. \tag{30}$$

6.2 χPT to N^2L order

Assuming that nuclear matter at ordinary density is a Fermi-liquid fixed point, one can look at the fluctuation in the strange flavor direction by using χPT. This has been recently worked out to N^2L order – one-loop in free space and two-loop order in medium – by Lee *et al.*[38].

There are several issues involved in this calculation.

- The first is to describe KN scattering at low energy in terms of χPT. To the order considered, there is no difficulty in doing this.

- The second is to extend the amplitude to off-shell. Here on-shell data fit in the first step are not enough to fix all the parameters of the Lagrangian. In fact there are two unknown quantities in the counter terms, one at next-to-leading order and the other at N^2L order. The first can be handled by assuming that the counter term is saturated by the decuplet resonances – which seems to be a reasonable thing to do. Therefore we are left with one free parameter. However appearing at subleading (N^2L) order, its uncertainty does not affect the calculation for small kaon frequency which is involved for kaon condensation.

- The third issue is to take into account many-body effects that figure in kaon-nuclear interactions entering in kaon condensation. Here not only effects on the kaon-nucleon amplitude embedded in the medium but also intrinsic many-body processes associated with n-Fermi interactions (for $n \geq 4$) in the chiral Lagrangian have to be treated. For

this the recent kaonic atom data[39] play an essential role. It turns out that *all* the parameters in the chiral Lagrangian can be fixed by the available kaonic atom data, allowing an almost parameter-free prediction for the critical density.

The result is

$$2 \lesssim \frac{\rho_c}{\rho_0} \lesssim 4 \tag{31}$$

in the same range as what is predicted in the skyrmion model. The lower limit is obtained when Brown-Rho scaling is implemented in the mean-field (up to $O(Q^2)$) terms. More details can be found in my Kyoto talk[40].

Acknowledgments

I would like to thank G.E. Brown, K. Kubodera, C.-H. Lee, H.K. Lee, D.-P. Min, M.A. Nowak, T.-S. Park, N.N. Scoccola, S.-J. Sin and I. Zahed for collaborations and discussions.

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